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REVIEW

Introduction to the Finite Element Method. By C. S. DESAI and J. F. ABEL. Van Nostrand Reinhold, 1972. 477 pp. £8.70.

The finite element method is a technique for obtaining approximate solutions to problems described by differential equations: the region of interest is divided into elementary subregions in each of which the solution is approximated by some simple functional form. There is now a vast and rapidly growing literature on the subject, whose extent stems in part from the wide applicability of the method and in part from the variety of levels at which it can be approached. These range from the mathematically sophisticated to the physically intuitive and the present book is very much towards the latter end of the spectrum. The authors' main goal has been to produce a systematic, simplified summary of available knowledge directed towards the needs of undergraduate and practising engineers both to enable them to solve specific problems and also to serve as an introduction to the research literature.

The book is divided into three parts. The first, consisting of some forty pages, presents background material on matrix techniques, the basic equations of solid mechanics and variational methods. Over half is on solid mechanics and this constitutes a very valuable summary, but the other two chapters would seem to be altogether too terse. The solution of the algebraic equations arising from the finite element method is such an essential part of the method that to cover equilibrium, eigenvalue and propagation problems in a mere nine pages severely limits the depth of understanding that one can attain of the method as distinct from its application. On the other hand, the limited coverage of variational methods will not ease the task of those interested in extending the ideas to fields other than solid mechanics.

The heart of the book consists of the second part, covering the theory of the finite element method with chapters on "The basic component – a simple element", "The overall problem – an assemblage of elements", "Techniques for nonlinear analysis", and "Generalization of the theory". Although the mathematical basis of the method was set out by R. Courant in 1943 it aroused no interest among mathematicians and the present developments owe almost everything to the efforts of engineers for whom, as for the present authors, it "is an off-spring of framed-structure analysis" (p. 68). This then is the basis of the present treatment. Moreover, attention is largely concentrated on displacement models which are discussed very fully. Generalized co-ordinate forms, natural co-ordinate systems and isoparametric elements are clearly presented together with alternative formulations of the element stiffness matrices and load vectors. There are twenty pages of worked examples in the first chapter to ensure that all the concepts are thoroughly understood. The relative advantages of conforming or non-conforming elements and of mesh refinement or higher order elements are commented upon though only from an experimental and not an analytical viewpoint; in the authors' words (p. 154) "The sub-

division process is essentially an exercise of engineering judgement" and little basis is given on which to exercise this judgement. Similarly, convergence and compatibility requirements are presented essentially as rules to be followed rather than their basis analysed. The absence of any attempt to analyse the errors committed at each stage also detracts from the consideration of nonlinear problems where the essential difference between the errors in the incremental procedures and the iterative procedures are not made clear; incidentally, the latter are wrongly equated to Newton's method.

The chapter on generalizations is a disappointment. Because of the specialization of the main chapters to the displacement model for structural mechanics, a much fuller development of wider concepts and alternative formulations would have been useful before other applications are treated in the third part of the book. For example, the residual method is accorded half a page only and the name of Galerkin does not appear in either the subject or author index although the Galerkin procedure and its underlying theory are essential to a proper consideration of non-variational problems.

The third part on applications consists mainly of examples collected from the published literature and arranged in chapters covering "Structural mechanics", "Soil and rock mechanics", "Dynamics, including earthquake analysis", "Torsion, heat conduction, and seepage", "Thermoelasticity, consolidation, hydroelasticity", and "Miscellaneous applications". Only in this last chapter does one find examples covering ideal fluids, viscous fluids and oscillations in harbours and lakes. In all three cases, however, only time-independent problems are considered so that little extension of the methods used in the earlier applications is called for.

Overall the book represents a valuable survey of present ideas and methods particularly in engineering problems. It does not set out to analyse the methods to any depth nor to develop them with a view to extending them into new fields. For the intended audience of engineers wishing to solve problems within the present mainstream it should be very valuable especially in view of its wealth of well-presented examples. For those wishing to develop applications in fluid mechanics, for example, it makes easily accessible the techniques and experience developed in the structural mechanics and allied fields. This is no mean feat for the many references, which are collected at the end of each chapter and cross-referenced, are largely drawn from specialized sources. However, such workers will have to look elsewhere for the mathematical basis of developing the finite element method and for its application to such fields.

K. W. Morton